Statistics

Lecture 19

Feb 19-8:47 AM

Critical Value:

SG.21 & SG.22

$z_{\alpha/2}$

α Alpha

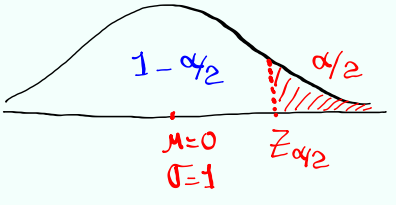
$0 < \alpha < 1$

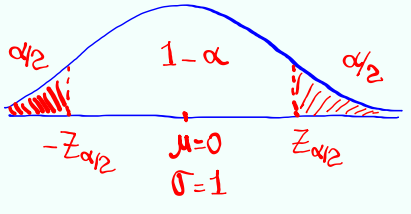
Significance level

$1 - \alpha$ Middle Area

$(1-\alpha)\cdot 100\%$ Confidence level

$z_{\alpha/2}$ is a value that separates the right tail with area $\alpha/2$ from the rest.





How to find $\pm z_{\alpha/2}$

2nd VARS

invNorm

Nov 10-10:37 AM

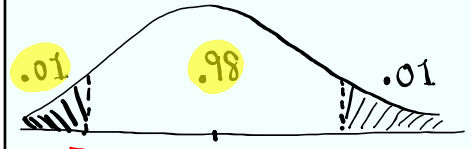
Find $z_{.01}$ → Right tail Area

$\alpha/2 = .01 \rightarrow \alpha = .02$ Significance level

$1 - \alpha = 1 - .02 = .98$ Middle Area

$.98 = .98(100)\%$

$= 98\%$ Confidence level



$-z_{.01}$ $\mu = 0$ $\sigma = 1$ $z_{.01} = \text{invNorm}(.99, 0, 1)$

-2.326 $= 2.326$

Nov 10-10:43 AM

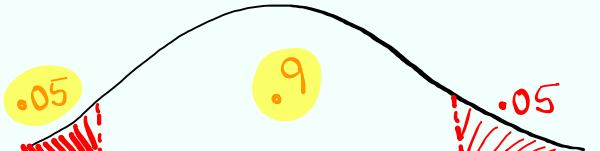
Find $\pm z_{\alpha/2}$ for $\alpha = .1$

$\alpha = .1$ Significance level

$\alpha/2 = .05$ Area of each tail

$1 - \alpha = .9$ Middle Area

$.9 = .9(100)\% = 90\%$ Confidence level



$-z_{.05}$ $\mu = 0$ $z_{.05} = \text{invNorm}(.95, 0, 1) = 1.645$

-1.645 $\sigma = 1$

Nov 10-10:49 AM

Find $\pm z_{\alpha/2}$ for 99% Confidence level

C- level

Middle Area = .99

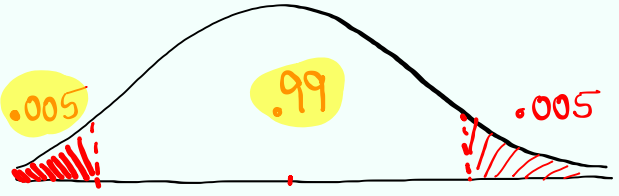
$\alpha/2 = .005$

Area on each tail

$1 - \alpha = .99$

$\alpha = .01$

Significance level



-2.576

$z_{.005}$ = invNorm(.995, 0, 1)

= 2.576

Nov 10-10:54 AM

$t_{\alpha/2}$ t - Dist.

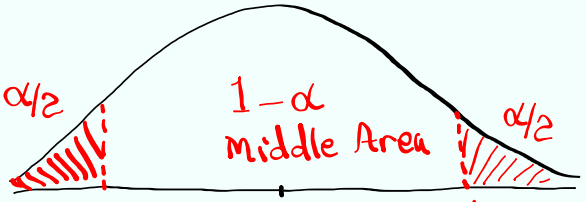
Symmetric, bell-shape with total area 1

mean = mode = median

$\mu = 0$, σ unknown

It comes with degrees of freedom

df



How to find $\pm t_{\alpha/2}$

2nd VARS

invT

Nov 10-11:00 AM

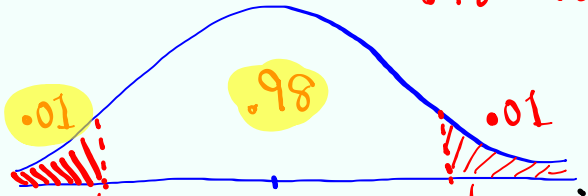
Find $t_{.01}$ with $df = 11$.

$t_{.01}$ → Area of the right tail

$\alpha/2 = .01 \rightarrow \alpha = .02$ Significance level

$1 - \alpha = 1 - .02 = .98$ Middle Area

$.98 = 98\%$ Confidence level



$t_{.01} = \text{invT}(.99, 11) = \boxed{2.718}$

Nov 10-11:06 AM

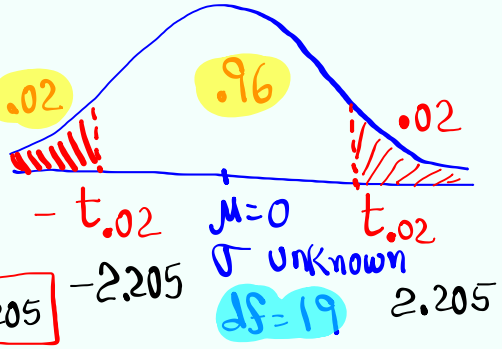
Find $\pm t_{\alpha/2}$ for $\alpha = .04$ and $df = 19$.

Sig. level $\alpha = .04$

Area on each tail $\alpha/2 = .02$

Middle Area $1 - \alpha = .96$

$.96 = 96\%$ Conf. level



$t_{.02} = \text{invT}(.98, 19) = \boxed{2.205}$

Nov 10-11:11 AM

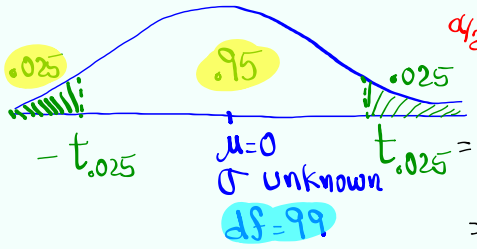
Find $\pm t_{\alpha/2}$ for 95% C-level & df = 99.

Middle Area = .95

$1 - \alpha = .95$

$\alpha = .05$ Sig. level

$\alpha/2 = .025$ Area of each tail



= invT(.975, 99)

= 1.984

As df increases,

$t_{\alpha/2} \approx Z_{\alpha/2}$

Nov 10-11:16 AM

Degrees of Freedom

It will be given to us or can be determined by topics.

20 students, and I bring 20 donuts.

First Student → 20 choices

Second " → 19 "

Third " → 18 "

⋮

Last Student → 0 choices (1 donut left)

df = 19

---

7 clean shirts, You will wear a clean shirt daily.

Monday 7 choices

Tuesday 6 "

Wednesday 5 "

⋮

Sunday 0 choice (1 clean shirt)

df = 6

Nov 10-11:22 AM

Estimating Parameters

SG 21

Estimation will be a range of values.

Confidence Interval

Every estimation comes with confidence level (C-level).

If C-level is not given, we use 95%.

Nov 10-11:29 AM

To estimate Population | we use Sample

- Proportion p | Proportion $\hat{p}$
- Mean μ | Mean $\bar{x}$
- Standard deviation σ | Standard deviation s

Point-estimate

Nov 10-11:32 AM

Estimating Population Proportion p

Answer $\quad < p <$

Format $\quad \hat{p} - E < p < \hat{p} + E$

$\hat{p}$ = p-hat = Sample Proportion

E = Margin of Error

$\hat{p} = \frac{x}{n}$ ← # of favorable responces; ← Sample Size

$\hat{q} = 1 - \hat{p}$

$E = z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}\,\hat{q}}{n}}$

$z_{\alpha/2}$ is the critical value for $(1-\alpha)\cdot 100\%$ C-level.

Nov 10-11:37 AM

I Surveyed 100 students and 80 of them had iPhone. $n = 100$ $\quad x = 80$

$\hat{p} = \frac{x}{n} = \frac{80}{100} = .8$

$\hat{q} = 1 - \hat{p} = .2$

Find 90% Confidence interval for the Prop. of all Students that have iPhone. C-level: .9

$\hat{p} - E < p < \hat{p} + E$

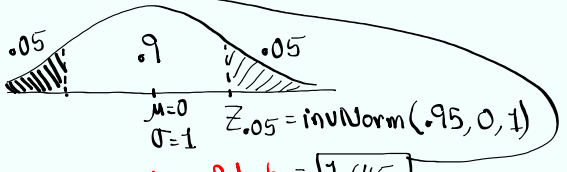
$E = z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}\hat{q}}{n}}$

$.8 - E < p < .8 + E$

$= 1.645 \cdot \sqrt{\frac{(.8)(.2)}{100}} \approx .07$

$.8 - .07 < p < .8 + .07$

$.73 < p < .87$



$= 1.645$

we are 90% confident that between 73% and 87% of all students have iphone

Nov 10-11:43 AM

Using TI:

STAT → TESTS → ↓ 1-PropZInt

$.73 < p < .87$

$E = \frac{.87 - .73}{2} = .07$

$\hat{p} = \frac{.87 + .73}{2} = .8$

x: 80
n: 100
C-level: .9
Calculate

Nov 10-11:53 AM

I surveyed 80 students, and 5% of them were Smokers.

$n = 80$
$\hat{p} = .05$ → $x = n\hat{p} = 80(.05) = 4$
if decimal → Round-up

Find 98% Conf. interval for the prop. of all Students that are Smokers.

C-level: .98

$-.01 < p < .11$

1-PropZInt
x = 4
n = 80
C-level: .98
Calculate

$E = \frac{.11 - (-.01)}{2} = .06$

$\hat{p} = \frac{.11 + (-.01)}{2} = .05$

Nov 10-11:58 AM

I surveyed 175 students and 8.2% of them were taking bus to get to school.

$n = 175$, $\hat{p} = .082$ $\rightarrow x = n\hat{p} = 175(.082) = 14.35 = 15$ Round-up

NO C-level $\Rightarrow$ use 95%

Find Confidence interval for proportion of all students that take buses to come to School.

1-PropZInt

$x = 15$

$n = 175$

C-level: .95

Calculate

$.04 < p < .13$

$E = \frac{.13 - .04}{2} = .045 \approx .05$

$\hat{p} = \frac{.13 + .04}{2} = .085 \approx 8.5\%$

Nov 10-12:08 PM

How to determine the minimum Sample Size: n

$$E = z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

Use Algebra to find n

$$n = \hat{p}\hat{q}\left(\frac{z_{\alpha/2}}{E}\right)^2$$

if decimal Always round-up

If $\hat{p}$ & $\hat{q}$ are unknown, we use .5 for each

$$n = .25\left(\frac{z_{\alpha/2}}{E}\right)^2$$

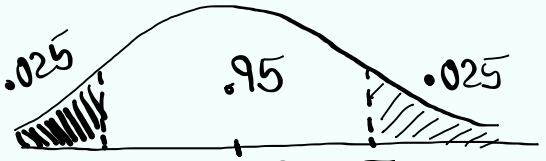
Nov 10-12:17 PM

Find minimum Sample Size needed to Construct 95% Conf. interval for pop. prop. if $\hat{p} = .3$ and $E = .05$

$$n = \hat{p}\cdot\hat{q}\left(\frac{z_{\alpha/2}}{E}\right)^2 = (.3)(.7)\left(\frac{1.960}{.05}\right)^2$$

$$= 322.6944$$

$$\approx \boxed{323}$$



$z_{.025} = \text{invNorm}(.975, 0, 1) = 1.960$

Nov 10-12:22 PM

I like to Construct 99% Conf. interval for prop. of all Students that Smoke and margin of error not to exceed 4%.

Find min. Sample Size needed.

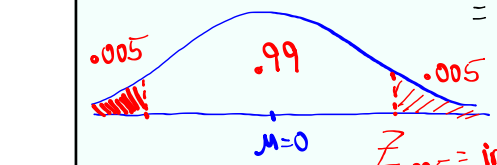
C-level: .99
E = .04
n = ?

$\hat{p}$ & $\hat{q}$ unknown

$$n = .25\left(\frac{z_{\alpha/2}}{E}\right)^2$$

$$= .25\left(\frac{2.576}{.04}\right)^2$$

$$= 1036.84 \approx \boxed{1037}$$



$z_{.005} = \text{invNorm}(.995, 0, 1) = \boxed{2.576}$

Nov 10-12:29 PM